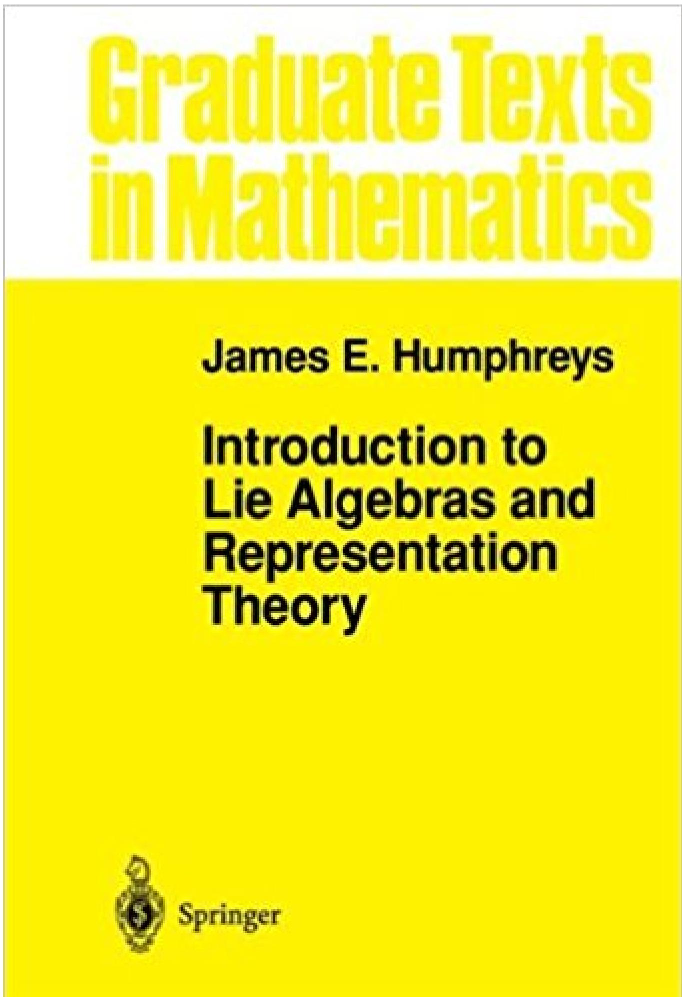
Introduction To Lie Algebras And Representation Theory (Graduate Texts In Mathematics) (v. 9)



Synopsis

This book is designed to introduce the reader to the theory of semisimple Lie algebras over an algebraically closed field of characteristic 0, with emphasis on representations. A good knowledge of linear algebra (including eigenvalues, bilinear forms, euclidean spaces, and tensor products of vector spaces) is presupposed, as well as some acquaintance with the methods of abstract algebra. The first four chapters might well be read by a bright undergraduate; however, the remaining three chapters are admittedly a little more demanding. Besides being useful in many parts of mathematics and physics, the theory of semisimple Lie algebras is inherently attractive, combining as it does a certain amount of depth and a satisfying degree of completeness in its basic results. Since Jacobson’s book appeared a decade ago, improvements have been made even in the classical parts of the theory. I have tried to incorÃ Â- porate some of them here and to provide easier access to the subject for non-specialists. For the specialist, the following features should be noted: (I) The Jordan-Chevalley decomposition of linear transformations is emphasized, with "toral" subalgebras replacing the more traditional Cartan subalgebras in the semisimple case. (2) The conjugacy theorem for Cartan subalgebras is proved (following D. J. Winter and G. D. Mostow) by elementary Lie algebra methods, avoiding the use of algebraic geometry.

Book Information

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Customer Reviews

J.E. HumphreysIntroduction to Lie Algebras and Representation Theory"An excellent introduction to the subject, ideal for a one semester graduate course."THE AMERICAN MATHEMATICAL MONTHLY"Exceptionally well written and ideally suited either for independent reading or as a text for an introduction to Lie algebras and their representations."MATHEMATICAL REVIEWSJ.E. Humphreys Introduction to Lie Algebras and Representation Theory "An excellent introduction to the subject, ideal for a one semester graduate course."???THE AMERICAN MATHEMATICAL MONTHLY "Exceptionally well written and ideally suited either for independent reading or as a text for an introduction to Lie algebras and their representations."???MATHEMATICAL REVIEWSJ.E. Humphreys Introduction to Lie Algebras and Representation Theory "An excellent introduction to the subject, ideal for a one semester graduate course."a "THE AMERICAN MATHEMATICAL MONTHLY "Exceptionally well written and ideally suited either for independent reading or as a text for an introduction to Lie algebras and their representations."a "MATHEMATICAL REVIEWS

Moves at an advanced pace, but doesn't skip any major steps in any arguments. The exercises make you think about the material.

The text on the subject - read it and work through the examples and you'll go far in life.

Professor Humphreys has accomplished clarification and teaching of this very core area of modern mathematics. He gives instructive examples and exercises.

Classical book about Lie algebras and Representaion Theory.

This book certainly leaves something to be desired, pedagogically speaking at least. Indeed, as other reviewers have noted, many of the proofs are unintuitive, and while concision is sometimes desirable, there are places in the book where the flow of the argument is mostly clear, assuming you're following along, rewriting the steps of the proof down on paper, and all of a sudden, seemingly out of nowhere, there is a single sentence condensing several steps of the argument, and you might find yourself pulling your hair out after hours of thinking about how to connect the dots. I think the textbook's problems can be summarized by the observation that there are (literally) almost no examples, and if there happens to be an example, none of the details are worked out. This is probably a good reference book, but, unless you have an excellent instructor, this is not a good book from which to learn the material for the first time. The problem is, there doesn't seem to be a genuinely pedagogical Lie algebras book as comprehensive as this one, and the interested student is left with few options. Can someone please fill this void?

This book is a pretty good introduction to the theory of Lie algebras and their representations, and its importance cannot be overstated, due to the myriads of applications of Lie algebras to physics, engineering, and computer graphics. The subject can be abstract, and may at first seem to have minimal applicability to beginners, but after one gets accustomed to thinking in terms of the representations of Lie algebras, the resulting matrix operations seem perfectly natural (and this is usually the approach taken by physicists). The book is aimed at an audience of mathematicians, and there is a lot of material covered, in spite of the size of the book. Readers who desire an historical approach should probably supplement their reading with other sources. Readers are expected to have a strong background in linear and abstract algebra, and the book as a textbook is geared toward graduate students in mathematics. Only semisimple Lie algebras over algebraically closed fields are considered, so readers interested in Lie algebras over prime characteristic or infinite-dimensional Lie algebras (such as arise in high energy physics), will have to look elsewhere. Physicists can profit from the reading of this book but close attention to detail will be required. The first chapter covers the basic definitions of Lie algebras and the algebraic properties of Lie algebras. No historical motivation is given, such as the connection of the theory with Lie groups, and Lie algebras are defined as vector spaces over fields, and not in the general setting of modules over a commutative ring. The four classical Lie algebras are defined, namely the special linear, symplectic, and orthogonal algebras. The physicist reader should pay attention to the (short) discussion on Lie algebras of derivations, given its connection to the adjoint representation and its importance in applications. The important notions of solvability and nilpotency are covered in fairly good detail. Engel's theorem, which essentially says that if all elements of a Lie algebra are nilpotent under the 'bracket", then the Lie algebra itself is nilpotent, is proven. The second chapter gives more into the structure of semisimple Lie algebras with the first result being the solution of the "eigenvalue" problem for solvable subalgebras of gl(V), where V is finite-dimensional. Cartan's criterion, giving conditions for the solvability of a Lie algebra, is proven, along with the criterion of semisimplicity using the Killing form. The representation theory of Lie algebras is begun in this chapter, with proof of Weyl's theorem. This theorem is essentially a generalization to Lie algebras of a similar result from elementary linear algebra, namely the Jordan decomposition of matrices. Again, physicist readers should pay close attention to the details of the discussion on root space decompositions. This is followed in chapter 3 by an in-depth treatment of root systems, wherein a positive-definite symmetric bilinear form is chosen on a fixed Euclidean space. These root systems enable a more transparent approach to the representation theory of Lie algebras. The theory of weights along with the Weyl group, allow a description of the representation theory that depends only on the root system. In addition, one can prove that two semisimple Lie algebras with the same root system are isomorphic, as is done in the next chapter. More precisely, it is shown that a semisimple Lie algebra and a maximal toral subalgebra is determined up to isomorphism by its root system. These maximal toral subalgebras are conjugate under the automorphisms of the Lie algebra. The author further shows that for an arbitary Lie algebra that is true, if one replaces the maximal toral subalgebra by a Cartan subalgebra. The proofs given do not use algebraic geometry, and so they are more accessible to beginning students. In chapter 5, the author introduces the universal enveloping algebra, and proves the Poincare-Birkhoff-Witt theorem. The goal of the author is to find a presentation of a semisimple Lie algebra over a field of characteristic 0 by generators and relations which depend only on the root system. This will show that a semisimple Lie algebra is completely determined by its root system (even if it is infinite dimensional). Chapter 6 is very demanding, and will require a lot of time to get through for the newcomer to the representation theory of Lie algebras. Weight spaces and maximal vectors are introduced in the context of modules over semisimple Lie algebras L. Finite dimensional irreducible L-modules are studied by first considering L-modules generated by a maximal vector. It is shown that if two standard cyclic modules of highest weight are irreducible, then they are isomorphic. The existence of a finite dimensional irreducible standard cyclic module is shown. Freudenthal's formula, which gives a formula for the multiplicity of an element of an irreducible L-module of heighest weight, is proven. A consideration of characters on infinite-dimensional modules leads to a proof of Weyl's formulas on characters of finite dimensional modules. The last chapter of the book considers Chevelley algebras and groups. Their introduction is done in the context of constructing irreducible integral representations of semisimple Lie algebras.

Humphreys' book on Lie algebras is rightly considered the standard text. Very thorough, covering the essential classical algebras, basic results on nilpotent and solvable Lie algebras, classification, etc. up to and including representations. Don't let the relatively small number of pages fool you; the book is quite dense, and so even covering the first 30 pages is a nice accomplishment for a student. Small caveat, the notation might be a bit confusing until you get used to it, but this is a common problem due to having both a Lie and a matrix product floating around, and is not a fault of the text. There is also a nice selection of exercises, between 5 and 10 per section.Highly recommended; every mathematician should know the basics of Lie algebras.

I must admit, my progress through this book can be measured in lines. It’s not that it’s confusing, but that it’s pretty dense. The proofs are structured in such a way as to leave teasing amount of details to the reader, and the text measures understanding as much as the exercises. It is that which makes reading this book worthwhile.From an academic point of view, the material in this book is very standard. The content of the first four chapters is closely paralleled by an introductory graduate level course in Lie Algebra and Representation Theory at MIT (although the instructor did not explicitly declare this as class text.) In many ways, this book is my ticket out of attending lectures, and it has done a great job so far.I must admit that it can be frustrating at times to work out the statements of the proofs, but it only makes the understanding just that much more pleasant and adds the perfect amount of emotion to an otherwise black/white text.

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